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Gauged models of multiflavoured chiral bosons

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Abstract : Interacting models of n_1 chiral bosons and n_2 gauge bosons in two dimensions are considered. The constraint structures are analysed and the physical spectra determined for two regularizations which lead to Lorentz invariant theories. The gauge bosons become massive in both cases. The number of unconfined chiral bosons is n_1 or n_1-n_2 depending on the regularization.

Keyword : Gauged chiral boson

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1. Introduction

A massless fermion field in two dimensional space time can be expressed in terms of a massless bosonic field. If one considers a chiral fermion, the equivalent is what is called a chiral boson. Chiral bosons have been widely investigated in recent years [1–6]. One of the most peculiar features of these fields is that the theory has to be described by a lagrangian density which is not manifestly Lorentz invariant unless auxiliary fields are introduced [2]. A consequence is that the introduction of gauge coupling is not straightforward. Three types of interactions have been proposed in the literature [3,5]. But one of these has turned out to be unacceptable [4,6], so that there are two known ways of introducing a gauge coupling. These give rise to field theoretic models involving a chiral boson coupled to a gauge boson. In the present paper, generalizations to several chiral bosons and several gauge bosons are worked out.

The first way of introducing a gauge coupling for chiral bosons comes from the two dimensional chiral Schwinger model [7], as formulated in terms of a chiral boson [8]. This model has the lagrangian density

$$\mathcal{L}_0^1 = (\dot\phi - \phi')\phi' + 2e\phi'(A_0 - A_1) - \frac{1}{2}e^2(A_0 - A_1)^2 + \frac{1}{2}ae^2\left(A_0^2 - A_1^2\right) + \frac{1}{2}F_{01}^2, \tag{1}$$

where e is the coupling and a is the regularization parameter. The spectrum of the theory contains a chiral boson and a massive boson, *i.e.* the gauge boson acquires mass. An alternative regularization of formally the same model yields the lagrangian density [5].

$$\mathcal{L} = (\dot{\phi} - \phi')\phi' + 2e\phi'(A_0 - A_1) - 2e^2 A_1^2 + \frac{1}{2}F_{01}^2, \tag{2}$$

which gives the second way of introducing a gauge coupling. Unlike the lagrangian (1) there is no explicit regularization parameter in (2), so this one corresponds to a single isolated model whereas the earlier one corresponds to a class of models. The difference in the two lies only in terms involving the gauge field arising from regularization. The spectrum of the second model contains only a massive boson, so that the chiral boson or the Weyl boson is confined.

2. Generalizations of interacting model-1

As mentioned above we shall consider generalizations of these models with several chiral bosons and several gauge fields. The continuous class of models (1) is generalized to

$$\begin{aligned}\mathcal{L}^{1} = {} & \left(\dot{\phi}_a - \phi_a'\right)\phi_a' + 2e_{\alpha a}\phi_a'\left(A_0^\alpha - A_1^\alpha\right) - \frac{1}{2}M_{\alpha\beta}\left(A_0^\alpha - A_1^\alpha\right)\left(A_0^\beta - A_1^\beta\right) \\ & + \frac{1}{2}N_{\alpha\beta}\left(A_0^\alpha A_0^\beta - A_1^\alpha A_1^\beta\right) + \frac{1}{2}F_{01}^\alpha F_{01}^\alpha .\end{aligned} \tag{3}$$

Here a takes the values 1, 2, ... n_1, α, β take the values 1, 2, ... n_2, $M_{\alpha\beta} = e_{\alpha a}e_{\beta a}$ and N is a symmetric matrix which is assumed to commute with $M_{\alpha\beta}$. To analyse the model, we let S be the orthogonal matrix that diagonalizes the symmetric matrices M and N :

$$\left(S^T M S\right)_{\alpha\beta} = d_\alpha \delta_{\alpha\beta}, \quad \left(S^T N S\right)_{\alpha\beta} = a_\alpha d_\alpha \delta_{\alpha\beta} \quad (\alpha \text{ not summed}),$$

where d_α and $a_\alpha d_\alpha$ are the (non-negative) eigenvalues of M and N. We use the transformation S to construct

$$\tilde{A}_\mu^\alpha = S_{\beta\alpha}A_\mu^\beta \tag{4a}$$

$$\tilde{e}_{\alpha a} = S_{\beta\alpha}e_{\beta a}. \tag{4b}$$

The lagrangian (3) can be written in terms of the transformed fields as

$$\begin{aligned}\mathcal{L}^{1T} = {} & \left(\dot{\phi}_a - \phi_a'\right)\phi_a' + 2\tilde{e}_{\alpha a}\left(\tilde{A}_0^\alpha - \tilde{A}_1^\alpha\right)\phi_a' - \frac{1}{2}\sum_\alpha d_\alpha\left(\tilde{A}_0^\alpha - \tilde{A}_1^\alpha\right)^2 \\ & + \frac{1}{2}\sum_\alpha a_\alpha d_\alpha\left(\left(\tilde{A}_0^\alpha\right)^2 - \left(\tilde{A}_1^\alpha\right)^2\right) + \frac{1}{2}\tilde{F}_{01}^\alpha \tilde{F}_{01}^\alpha .\end{aligned} \tag{5}$$

We shall drop the tildes for convenience. The canonical momenta corresponding to ϕ_a, A_0^α and A_1^α are respectively

$$\pi_a = \phi'_a, \tag{6a}$$

$$\pi_0^\alpha = 0, \tag{6b}$$

$$\pi_1^\alpha = F_{01}^\alpha. \tag{6c}$$

The first two sets among these are primary constraints. The canonical Hamiltonian is found to be

$$H^1 = \int dx \Big[\frac{1}{2}\pi_1^\alpha \pi_1^\alpha + \pi_1^\alpha A_0'^\alpha + (\phi'_a)^2 - 2e_{\alpha a}\phi'_a (A_0^\alpha - A_1^\alpha) + \frac{1}{2}\sum_\alpha d_\alpha (A_0^\alpha - A_1^\alpha)^2 - \frac{1}{2}\sum_\alpha a_\alpha d_\alpha \Big((A_0^\alpha)^2 - (A_1^\alpha)^2 \Big) \Big] = \int dx \mathcal{H}^1 \tag{7}$$

The primary constraints have to be preserved in time, and this leads to the secondary constraint

$$\pi_1'^\alpha + 2e_{\alpha a}\phi'_a + d_\alpha \big[(a_\alpha - 1)A_0^\alpha + A_1^\alpha \big] = 0. \tag{8}$$

The three sets of constraints (6a), (6b) and (8) are second class if it is assumed that each a_α differs from unity and each d_α differs from zero. They may be used to solve for π_a, π_0^α and A_0^α in terms of the other fields. The Dirac brackets between these fields are canonical except in the case of ϕ_a:

$$\{\phi_a(x), \phi_b(y)\}^* = -\frac{1}{4}\delta_{ab}\varepsilon(x-y). \tag{9}$$

The Hamiltonian rewritten in terms of the fields ϕ_a, A_1^α and π_1^α becomes

$$H_r^1 = \int dx \Big[\frac{1}{2}\pi_1^\alpha \pi_1^\alpha + \frac{1}{2(a_\alpha - 1)d_\alpha}\pi_1'^\alpha \pi_1'^\alpha + 2\frac{e_{\alpha a}}{d_\alpha (a_\alpha - 1)}\pi_1'^\alpha \phi'_a + (\phi'_a)^2 + 2V_{ab}\phi'_a\phi'_b + \frac{\pi_1'^\alpha A_1^\alpha}{(a_\alpha - 1)} + 2\frac{e_{\alpha a}}{(a_\alpha - 1)}\phi'_a A_1^\alpha + \frac{d_\alpha a_\alpha}{2(a_\alpha - 1)}(A_1^\alpha)^2 \Big] = \int dx \mathcal{H}_r^1, \tag{10}$$

where $$V_{ab} = \sum_\alpha \frac{e_{\alpha a}e_{\alpha b}}{a_\alpha - 1}.$$

The equations of motion arising from this Hamiltonian (10) can be simplified by introducing

$$\bar{\phi}_\alpha = \frac{e_{\alpha a}\phi_a}{\sqrt{d_\alpha}}, \tag{11a}$$

$$\bar{\phi}_A = T_{Aa}\phi_a. \tag{11b}$$

Here, T_{Au} satisfies the conditions : $T_{Au}e_{\alpha u} = 0$ for each α, $T_{Au}T_{Bu} = \delta_{AB}$. The number of allowed values of A is obviously $n_1 - n_2$. One finds the equations

$$\dot{\bar{\phi}}_A = \bar{\phi}'_A, \tag{12a}$$

$$\dot{\bar{\phi}}_\alpha = \frac{a_\alpha + 1}{a_\alpha - 1}\bar{\phi}'_\alpha + \frac{a_\alpha\sqrt{d_\alpha}}{(a_\alpha - 1)}A_1 + \frac{1}{\sqrt{d_\alpha}(a_\alpha - 1)}\pi_1'^\alpha \tag{12b}$$

$$A_1^\alpha = \pi_1^{\alpha\cdot} - \frac{\pi''^\alpha}{d_\alpha(a_\alpha - 1)} - 2\frac{\bar{\phi}''_\alpha}{\sqrt{d_\alpha}(a_\alpha - 1)} - \frac{1}{(a_\alpha - 1)}A_1'^\alpha, \tag{12c}$$

$$\dot{\pi}_1^\alpha = -\frac{\pi_1'^\alpha}{(a_\alpha - 1)} - 2\frac{\sqrt{d_\alpha}}{a_\alpha - 1}\phi_\alpha - \frac{d_\alpha a_\alpha^2}{(a_\alpha - 1)}A_1^\alpha. \tag{12d}$$

The last three sets of equations can be easily solved in terms of the free fields h_α and σ_α which satisfy

$$h_\alpha = h'_\alpha, \tag{13a}$$

$$\left(\Box + \frac{d_\alpha a_\alpha^2}{(a_\alpha - 1)}\right)\sigma_\alpha = 0, \tag{13b}$$

$$\bar{\phi}_\alpha = \sigma_\alpha - h_\alpha \tag{13c}$$

$$A_{\alpha 1} = \frac{1}{a_\alpha\sqrt{d_\alpha}}\left[(a_\alpha - 1)\dot{\sigma}_\alpha - \sigma'_\alpha + \partial + h_\alpha\right]. \tag{13d}$$

Thus, there are n_2 massive bosons with masses $\sqrt{\frac{a_\alpha d_\alpha}{a_\alpha - 1}}$, and n_1 chiral bosons of which n_2 are the h_α and the remaining $n_1 - n_2$ are the $\bar{\phi}_A$.

It is interesting that the spectrum is relativistically invariant though the lagrangian itself is not Lorentz invariant. The Poincaré algebra is easily checked as follows. The reduced Hamiltonian has already been given in (10). The canonical momentum is

$$p^1 = \int dx\left[\pi_u\phi'_u + \pi_1^\alpha A_1'^\alpha + \pi_0^\alpha A_0^\alpha\right]. \tag{14}$$

Introduction of the constraints reduces this to

$$p_r^1 = \int dx\left[(\phi'_u)^2 + \pi_1^\alpha A_1'^\alpha\right]. \tag{15}$$

The canonical boost generator

$$M^1 = \int dx\left[x\mathcal{H}^1 + \pi_1^\alpha A_0^\alpha + \pi_0^\alpha A_1^\alpha\right] + tp^1 \tag{16}$$

similarly reduces to

$$M_r^1 = tp_r^1 + \int dx\left[x\mathcal{H}_r^1 - \frac{\pi_1^\alpha\pi_1'^\alpha + 2e_{\alpha u}\phi'_u\pi_1^\alpha + \pi_1^\alpha A_1^\alpha}{a - 1}\right]. \tag{17}$$

Straightforward calculation shows that

$$\left\{M^1_r,\, P^1_r\right\}^* = -H^1_r, \tag{18a}$$

$$\left\{M^1_r,\, H^1_r\right\}^* = -P^1_r. \tag{18b}$$

Using Dirac brackets the Lorentz transformations on the fields are found to be as follows :

$$\left\{A^\alpha_1,\, M^1_r\right\}^* = x\dot{A}^\alpha_1 + tA'^\alpha_1 - \frac{1}{a_\alpha - 1}A^\alpha_1 - \frac{2e_{\alpha a}}{d_\alpha(a_\alpha - 1)}\phi'^a, \tag{19a}$$

$$\left\{\pi^\alpha_1,\, M^1_r\right\}^* = x\dot{\pi}^\alpha_1 + t\pi'^\alpha_1 + \frac{1}{a_\alpha - 1}\pi^\alpha_1, \tag{19b}$$

$$\left\{\phi^a,\, M^1_r\right\}^* = x\dot{\phi}^a + t\phi'^a - \frac{e_{\alpha a}}{d_\alpha(a_\alpha - 1)}A'^\alpha_1. \tag{19c}$$

As the theory is Lorentz-invariant, one can write a Lorentz invariant lagrangian density [4] for the above lagrangian (5) as

$$\mathcal{L}^1_U = \frac{1}{2}\left((\dot{\phi}_a)^2 - (\phi'_a)^2\right) + 2\tilde{e}_{\alpha a}(\dot{\phi}_a + \phi'_a)\left(\tilde{A}^\alpha_0 - \tilde{A}^\alpha_1\right) + \frac{1}{2}\tilde{F}^\alpha_{01}\tilde{F}^\alpha_{01}$$
$$+ \frac{1}{2}a_\alpha d_\alpha\left(\left(\tilde{A}^\alpha_0\right)^2 - \left(\tilde{A}^\alpha_1\right)^2\right) + \frac{1}{2}\lambda\left(\dot{\phi}_a - \phi'_a + \tilde{e}_{\alpha a}\left(\tilde{A}^\alpha_0 - \tilde{A}^\alpha_1\right)\right)^2. \tag{20}$$

For convenience, we shall drop the tildes. This lagrangian involves one auxiliary field λ. The canonical momenta conjugate to the fields $\phi^a, A^\alpha_1, A^\alpha_0$ and λ are respectively

$$\pi_a = (1+\lambda)\dot{\phi}_a - \lambda\phi'_a + (1+\lambda)e_{\alpha a}\left(A^\alpha_0 - A^\alpha_1\right), \tag{21a}$$

$$\pi^\alpha_1 = F^\alpha_{01}, \tag{21b}$$

$$\pi^\alpha_0 = 0, \tag{21c}$$

$$\pi_\lambda = 0. \tag{21d}$$

The Hamiltonian density is found to be

$$\mathcal{H}^1 = \frac{\pi^2_a + (\phi'_a)^2}{2(1+\lambda)} + \frac{\pi_a\left[\lambda\phi_a - (1+\lambda)e_{\alpha a}\left(A^\alpha_0 - A^\alpha_1\right)\right]}{1+\lambda} + \frac{\left(\pi^\alpha_1\right)^2}{2}$$
$$+ \pi^\alpha_1 A'^\alpha_0 - e_{\alpha a}\phi'_a\left(A^\alpha_0 - A^\alpha_1\right) + \frac{1}{2}d_\alpha\left(A^\alpha_0 - A^\alpha_1\right)^2 - \frac{1}{2}a_\alpha d_\alpha$$
$$\times\left(\left(A^\alpha_0\right)^2 - \left(A^\alpha_1\right)^2\right) + v^\alpha_0\pi^\alpha_0 + v_1\pi_\lambda, \tag{22}$$

where ν_0 and ν_1 are velocities not yet determined. The conservation of the primary constraints (21c) and (21d) lead to the following two secondary constraints

$$\pi_1'^{\alpha} + e_{\alpha u}(\pi_u + \phi_u') + d_\alpha\left((a_\alpha - 1) A_0^\alpha + A_1^\alpha\right) = 0, \tag{23a}$$

$$\pi_u - \phi_u' = 0. \tag{23b}$$

The constraint (21c) is fixed for all times by fixing the velocity ν_0^α. The constraint (21d) remains first class. This means ν_1 is undetermined. One gauge condition is to be chosen to make the constraint (21d) second class. A suitable choice is

$$\lambda = \text{constant}, \tag{24a}$$

which yields

$$\nu_1 = 0. \tag{24b}$$

On insertion of (23b), (24b) in (22), the auxiliary field λ drops out. The constraints (21c), (23a) and (23b) are equivalent to the set of constraints (6a), (6b) and (8). In the reduced phase space described by $(\phi_{u'}, \pi_u)$, $(A_1^\alpha, \pi_1^\alpha)$ and $(A_0^\alpha, \pi_0^\alpha)$ the Hamiltonian density $\mathcal{H}^{-1}$ is found to be equal to the Hamiltonian density in (7).

3. Generalizations of interacting model-2

Next we consider the generalization of the isolated model (2), described by the lagrangian density

$$\mathcal{L}^2 = (\dot\phi_u - \phi_u')\phi_u' + 2e_{\alpha u}\phi_u'\left(A_0^\alpha - A_1^\alpha\right) - M_{\alpha\beta}A_1^\alpha A_1^\beta + \frac{1}{2}F_{01}^\alpha F_{01}^\alpha \tag{25}$$

As before, $M_{\alpha\beta} = e_{\alpha u}e_{\beta u}$. By carrying out the same transformation (4a) and (4b) as before, we can diagonalize M. Thus, it is sufficient to consider the lagrangian density

$$\mathcal{L}^{2T} = (\dot\phi_u - \phi_u')\phi' + 2e_{\alpha u}\phi_u'\left(A_0^\alpha - A_1^\alpha\right) - 2d_\alpha A_1^\alpha A_1^\alpha + \frac{1}{2}F_{01}^\alpha F_{01}^\alpha. \tag{26}$$

The canonical momenta corresponding to ϕ_u, A_0^α, A_1^α are respectively

$$\pi_u = \phi_u', \tag{27a}$$

$$\pi_0^\alpha = 0, \tag{27b}$$

and

$$\pi_1^\alpha = F_{01}^\alpha. \tag{27c}$$

The two constraints (27a) and (27b) are exactly the same as before. The canonical Hamiltonian is

$$H^2 = \int dx\left[\frac{1}{2}\pi_1^\alpha\pi_1^\alpha + \pi_1^\alpha A_0'^\alpha + (\phi_u')^2 - 2e_{\alpha u}\phi_u'\left(A_0^\alpha - A_1^\alpha\right) + 2d_\alpha A_1^\alpha A_1^\alpha\right] = \int dx\,\mathcal{H}^2 \tag{28}$$

The requirement that the primary constraints remain unaltered in time, leads to two sets of secondary constraints

$$\pi_1'^{\alpha} + 2e_{\alpha a}\phi_a' = 0, \tag{29a}$$

$$A_0'^{\alpha} + A_1'^{\alpha} = 0. \tag{29b}$$

Using the four sets of constraints which are second class, it is possible to eliminate A_0^{α}, π_0^{α}, π_1^{α} and π_a. The Dirac brackets between the remaining variables are

$$\left\{A_1^{\alpha}(x), \phi_a(y)\right\}^* = -\frac{e_{\alpha a}}{2d_{\alpha}}\delta(x-y), \tag{30a}$$

$$\left\{A_1^{\alpha}(x), A_1^{\alpha}(y)\right\}^* = -\frac{1}{2d_{\alpha}}\delta'(x-y), \tag{30b}$$

$$\left\{\phi_a(x), \phi_b(y)\right\}^* = -\frac{1}{4}Z_{ab}\varepsilon(x-y), \tag{30c}$$

where $\quad Z_{ab} = \delta_{ab} - 4e_{\alpha a}e_{\alpha b}.$

The Hamiltonian, rewritten using the constraints, is

$$\begin{aligned} H_r^2 &= \int dx\left[\phi_a'\phi_a' + 2e_{\alpha a}\phi_a'A_1^{\alpha} + 2d_{\alpha}A_1^{\alpha}A_1^{\alpha} + 2U_{ab}\phi_a\phi_b\right] \\ &= \int dx\mathcal{H}_r^2 \end{aligned} \tag{31}$$

where $\quad U_{ab} = e_{\alpha a}e_{\alpha b}.$

The equations of motion produced by this Hamiltonian are

$$\dot{\phi}_a = \phi_a' + 2e_{\alpha a}A_1^{\alpha}, \tag{32a}$$

$$\dot{A}_1^{\alpha} = -A_1'^{\alpha} - 2e_{\alpha a}\phi_a. \tag{32b}$$

Once again, we introduce $\tilde{\phi}_{\alpha}$ and $\bar{\phi}_A$ to obtain the equations

$$\dot{\bar{\phi}}_A = \bar{\phi}_A', \tag{33a}$$

$$(\Box + 4d_{\alpha})A_1^{\alpha} = (\Box + 4d_{\alpha})\tilde{\phi}_{\alpha} = 0. \tag{33b}$$

The $\bar{\phi}_A$ constitute n_1–n_2 chiral bosons while A_1^{α} and $\tilde{\phi}_{\alpha}$ are combinations of n_2 massive boson fields and their momenta.

The spectrum is Lorent-invariant again though (26) is not. This model also satisfies usual Poincaré algebra. The canonical momentum and the boost generator are given by

$$P^2 = \int dx\left[\pi_a\phi_a' + \pi_1^{\alpha}A_1^{\alpha} + \pi_0^{\alpha}A_0^{\alpha}\right], \tag{34}$$

$$M^2 = \int dx\left[x\mathcal{H}^1 + \pi_1^\alpha A_0^\alpha + \pi_0^\alpha A_1^\alpha\right] + tP^2 \tag{35}$$

On introducing the constraints, the two reduce to the following :

$$P_r^2 = \int dx\left[(\phi_u')^2 + \pi_1^\alpha A_1'^\alpha\right], \tag{36}$$

$$M_r^2 = tP_r + \int dx\left[x\mathcal{H}_r^2 - \pi_1^\alpha A_1^\alpha\right]. \tag{37}$$

A straightforward calculation shows that M_r^2 is conserved and generates the correct transformation on P_r^2 and H_r^2 *i.e.*

$$\left\{M_r^2, P_r^2\right\}^* = -H_r^2, \tag{38a}$$

$$\left\{M_r^2, H_r^2\right\}^* = -P_r^2. \tag{38b}$$

The Lorentz transformation for the fields are given by

$$\left\{A^\alpha, M_r^2\right\}^* = x\dot{A}_1^\alpha + tA_1'^\alpha - A_1^\alpha, \tag{39a}$$

$$\left\{\phi^u, M_r^2\right\}^* = x\dot{\phi}^u + t\phi'^u. \tag{39b}$$

Here also we can get a Lorentz invariant lagrangian density [6] which is equivalent to the lagrangian density (29) :

$$\begin{aligned}\mathcal{L}_U^2 = {} & \frac{1}{2}\left((\dot{\phi}_u)^2 - (\phi_u')^2\right) + 2\tilde{e}_{\alpha u}(\dot{\phi}_u + \phi_u')\left(\tilde{A}_0^\alpha - \tilde{A}_1^\alpha\right) + \frac{1}{2}\tilde{F}_{01}^\alpha\tilde{F}_{01}^\alpha \\ & + d_\alpha\left(\left(\tilde{A}_0^\alpha\right)^2 + \left(\tilde{A}_1^\alpha\right)^2\right) + \frac{1}{2}\lambda\left(\dot{\phi}_u - \phi_u' + \tilde{e}_{\alpha u}\left(\tilde{A}_0^\alpha - \tilde{A}_1^\alpha\right)\right)^2 \\ & + \frac{1}{2}\rho\left(\dot{\tilde{A}}_1^\alpha - \tilde{A}_0'^\alpha + 2\rho e_{\alpha u}\phi_u\right) .\end{aligned} \tag{40}$$

Here, instead of one auxiliary field as in lagrangian (20), two auxiliary fields are present. The canonical momenta are

$$\pi_u = (1+\lambda)\dot{\phi}_u - \lambda\phi_u' + (1+\lambda)e_{\alpha u}\left(A_0^\alpha - A_1^\alpha\right), \tag{41a}$$

$$\pi_1^\alpha = (1+\rho)F_{01}^\alpha + 2\rho e_{\alpha u}\phi_u, \tag{41b}$$

$$\pi_0^\alpha = 0, \tag{41c}$$

$$\pi_\lambda = 0, \tag{41d}$$

$$\pi_\rho = 0. \tag{41e}$$

The Hamiltonian density is found to be

$$\tilde{\mathcal{H}}^2 = \frac{\pi_a^2 + (\phi_a')^2}{2(1+\lambda)} + \frac{\pi_a\left[\lambda\phi_a - (1+\lambda)e_{\alpha a}(A_0 - A_1)\right]}{1+\lambda} + \frac{\left(\pi_1^\alpha\right)^2}{2}$$
$$+ \pi_1^\alpha A_0' - e_{\alpha a}\phi_a'(A_0 - A_1) + \frac{1}{2}d_\alpha(A_0 - A_1)^2 - d_\alpha\left(\left(A_0^1\right)^2 - \left(A_1^\alpha\right)^2\right)$$
$$+ \frac{\left(\pi_1^\alpha\right)^2}{2(1+\rho)} + \frac{\pi_1^\alpha\left[(1+\rho)A_0'^\alpha - 2\rho e_{\alpha a}\phi^a\right]}{(1+\rho)} - \frac{2\rho d_\alpha}{1+\rho}(\phi_a)^2$$
$$+ w_0^\alpha\pi_0^\alpha + w_1\pi_\lambda + w_2\pi_\rho, \tag{42}$$

where w_0, w_1, w_2 are the three velocities corresponding to the three primary constraints (41c), (41d) and (41e). For conservation of the primary constraints, the following three constraints appear :

$$\pi_1'^\alpha + e_{\alpha a}(\pi_a + \phi_a') + d_\alpha\left(A_0^\alpha + A_1^\alpha\right) = 0, \tag{43a}$$

$$\pi_a - \phi_a' = 0, \tag{43b}$$

$$\pi_1^\alpha + 2e_{\alpha a}\phi_a = 0. \tag{43c}$$

Out of these six constraints, two ((41d) and (41e)), are first class and the rest are second class. For the two first class constraints, two gauge conditions have to be chosen. With the choice λ = constant and ρ = constant, the velocities are determined to be

$$w_2 = 0 = w_1. \tag{44}$$

The eqs. (43a), (43b) and (44) make the Hamiltonian free from auxiliary fields. Here also the set of constraints (41c), (43a), (43b) and (43c) are equivalent to the set (27a), (27b), (29a) and (29b) and in the reduced phase space described by (ϕ_a, π_a), $(A_1^\alpha, \pi_1^\alpha)$ and $(A_0^\alpha, \pi_0^\alpha)$, the Hamiltonian density $\tilde{\mathcal{H}}^2$ is found to be identical to the Hamiltonian density in (28) of the Lorentz-noninvariant formalism.

4. Conclusions

The number of massive gauge bosons is the same in the two cases, but the number of chiral bosons left in the spectrum is different. Whereas in the first case all chiral bosons remain in the spectrum, in the second case some are confined, and all can be confined by choosing $n_1 = n_2$.

One of the most remarkable features of these models is the dependence of the extent of confinement on the regularization. It would be interesting to identify similar phenomena in four dimensions. In practical high energy physics, the problem of giving masses to gauge bosons is a real one. Although chiral bosons do not exist in four dimensions, it is worth trying to find out models where some analogues of chiral bosons give masses to gauge bosons and disappear from the spectrum in the process, *i.e.* are completely eaten up.

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